

PROGRESSION THROUGH CALCULATIONS FOR MULTIPLICATION

In Key Stage 1, the principal focus of mathematics teaching is to ensure that **all** children develop confidence and mental fluency with whole numbers, counting and place value. This will involve working with numerals, words and the four operations and will include the use of practical resources. By the end of year 2, children should know number bonds to 20 and use understanding of place value with numbers to at least 100.

In years 3 and 4, teaching will ensure that children become increasingly fluent with whole numbers, the four operations and place value. They will develop efficient written and mental methods and will work on calculations using increasingly large whole numbers.

The main focus for teaching in years 5 and 6 is to ensure that children extend their understanding of the number system and place value using large whole numbers (up to 10 000 000)

By the end of year 6, pupils should be fluent in written methods for all four operations and in working with fractions, decimals and percentages. They should multiply 4 digits by a 2 digit whole number using formal written method of long multiplication.

Children should not be made to go onto the next stage if:

- 1) They are not ready.
- 2) They are not confident.

Children should be encouraged to check their answers after calculation using an appropriate strategy. Children should also be encouraged to consider if a mental calculation would be more accurate and efficient before using written methods.

They are expected to use mathematical vocabulary and to read, spell and pronounce the language correctly.

MENTAL CALCULATIONS

(ongoing)

These are a **selection** of mental calculation strategies. Children are encouraged to use mental calculation strategies alongside written methods.

Repeated addition

Use as the main introduction to the concept of multiplication.

Using multiplication facts

Tables should be introduced from Y2 onwards.

Children must also understand the effect of multiplying by 0 and by 1.

Year 2 2 times table
 5 times table
 10 times table and others if appropriate

Year 3 2 times table
 3 times table
 4 times table
 5 times table
 8 times table
 10 times table and others if appropriate

Year 4 Derive and recall all multiplication facts up to 12×12

Years 5 & 6 Derive and recall **quickly** all multiplication facts up to 12×12

Doubling and halving

Knowing that doubling is multiplying by 2

Applying the knowledge of doubles and halves to known facts e.g. 8×4 is double 4×4

Multiplying by 10 or 100 or 1000 (Place value)

Knowing that the effect of multiplying by 10 is to move the digits one place to the left using zero as a place holder (10 times bigger).

Knowing that the effect of multiplying by 100 is to move the digits two places to the left using zero as a place holder (100 times bigger).

Knowing that the effect of multiplying by 1000 is to move the digits three places to the left using zero as a place holder (1000 times bigger).

Commutative rule

Multiplication can be done in any order (3×5 gives the same answer as 5×3).

Helps with calculations such as $4 \times 12 \times 5$. This can become $4 \times 5 \times 12$ or $20 \times 12 = 240$.

Partitioning

$$\begin{aligned} 23 \times 4 &= (20 \times 4) + (3 \times 4) \\ &= 80 + 12 \\ &= 102 \end{aligned}$$

Using and applying multiplication facts

Children should be able to utilise their tables knowledge to derive other facts.

e.g. If I know $3 \times 7 = 21$, what else do I know?

$7 \times 3 = 21$, $30 \times 7 = 210$, $300 \times 7 = 2100$, $3000 \times 7 = 21\,000$, $0.3 \times 7 = 2.1$ etc

Use of factors

$$8 \times 12 = 8 \times 4 \times 3$$

THE FOLLOWING ARE STRATEGIES THAT CHILDREN ARE EXPECTED TO USE AND UNDERSTAND.

INITIALLY

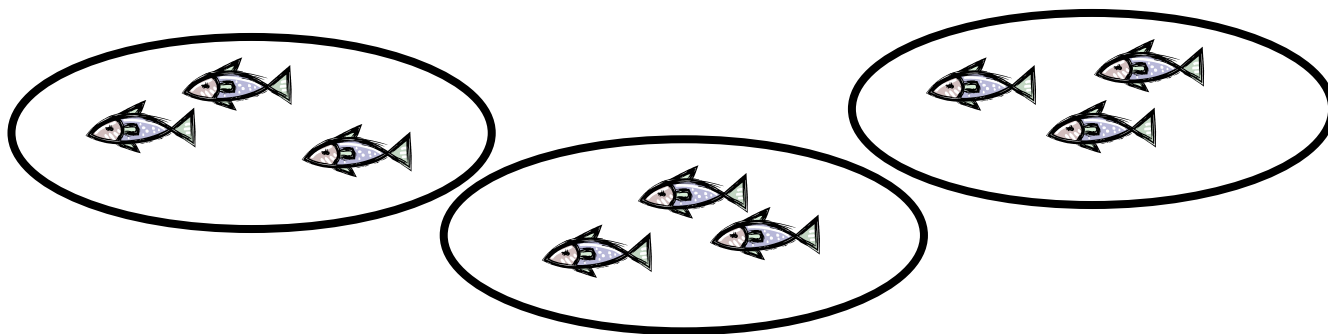
Through practical and play opportunities children should experience seeing multiple groups of the same number of objects (sets). They will understand that these sets can be combined to make a total.

THEN

Children should use a range of number lines; horizontal, vertical, blank and also number 'ladders' to support their understanding of multiplication (as repeated addition).

Children will experience equal groups of objects and will count in 2s and 10s and then move onto counting in 5s. Children should have experience of counting in multiples both forwards and backwards. They will work on practical problem solving activities involving equal sets or groups. **Money is especially helpful in the teaching of 2's, 5's and 10's and should regularly be used.**

Doubling numbers of items and halving again should increase children's understanding of the relationship between doubling and halving. Children should connect the use of the 10 times table to place value and the five times table to the divisions on the clock face.



NEXT

Children will develop their understanding of multiplication and use jottings to support calculations. They will be using \times and $=$ signs.

Visual representations and practical methods will be used extensively to ensure children understand the difference between eg 5×3 and 3×5 .

3 multiplied by 5 is $3 + 3 + 3 + 3 + 3 = 15$ or 5 lots of 3 or 3×5

NOT 5×3

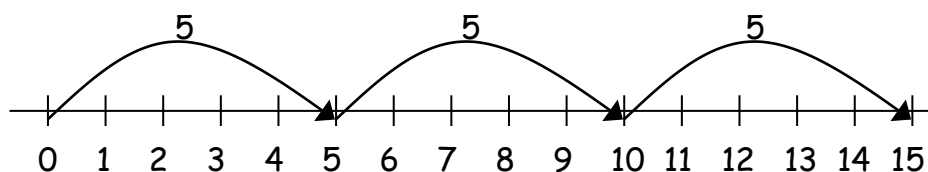
5 multiplied by 3 is $5 + 5 + 5 = 15$ or 3 lots of 5 or 5×3

NOT 3×5

Repeated addition

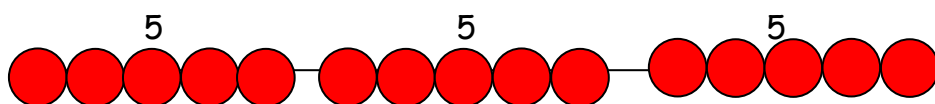
Repeated addition can be shown easily on a number line:

$$5 \times 3 = 5 + 5 + 5$$



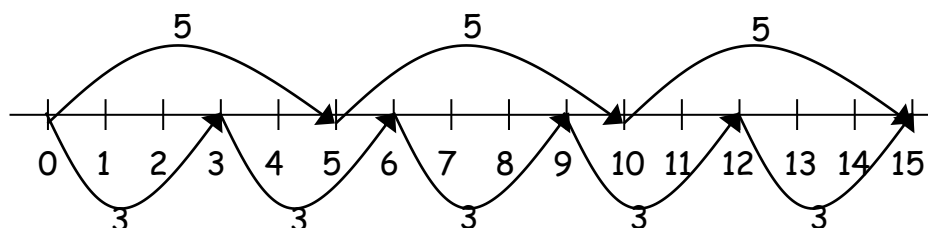
and on a bead string:

$$5 \times 3 = 5 + 5 + 5$$



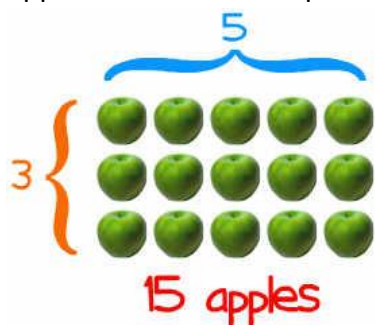
Commutativity

Children should know that 3×5 has the same answer as 5×3 . This can also be shown on the number line.

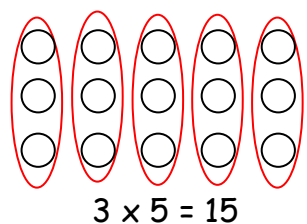


Arrays

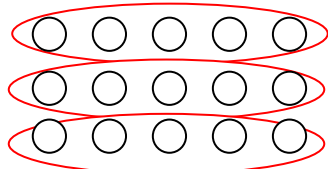
Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method. They begin with real items.



Then move to using dots or squares to represent items.



This is 5 lots of 3 so we write $3 \times 5 = 15$



This is 3 lots of 5 so we write $5 \times 3 = 15$

Arrays can show both calculations depending on which way they are looked at; either across or down.

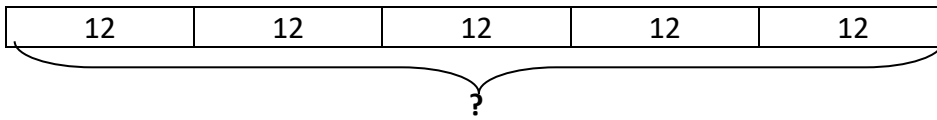
Use of arrays should be used to answer open-ended questions such as "How many arrays can you make using eg 12 counters?"

Bar Modelling

Children will be shown how to use bar modelling as a visual method to solve multiplication problems. They will move from using real objects, to representations of real objects. They are taught to recognise that each equal group is part of the whole.

5 boxes of pencils. There are 12 pencils in each box.

Calculation



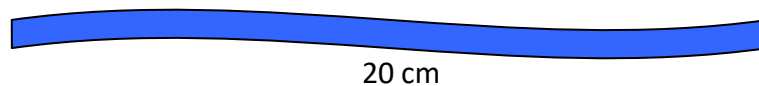
$$12 \times 5 =$$

AFTER THAT

Children will develop an understanding of

Scaling - Problem-solving

e.g. Find a ribbon that is 4 times as long as the blue ribbon



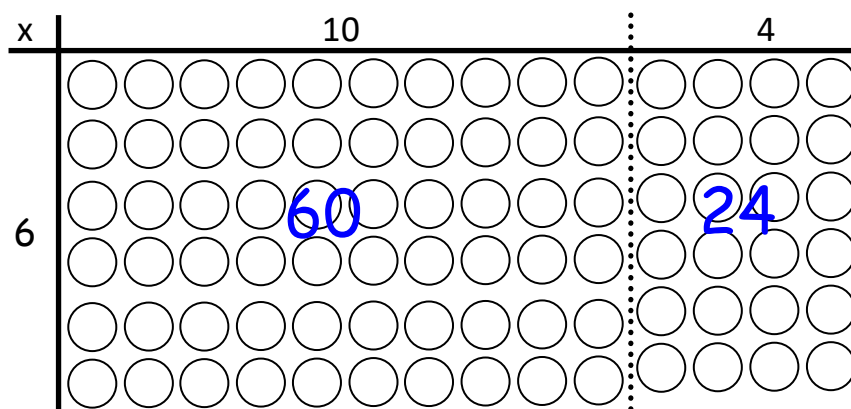
Using symbols to stand for unknown numbers to complete equations using inverse operations

$$\square \times 5 = 20$$

$$3 \times \triangle = 18$$

$$\diamond \times \square = 32$$

Partitioning – Initially use arrays to show how partitioning works with multiplication



$$(6 \times 10) + (6 \times 4)$$

$$60 + 24$$

$$84$$

Leading to:

$$\begin{aligned} 24 \times 6 &= (20 \times 6) + (4 \times 6) \\ &= 120 + 24 \\ &= 144 \end{aligned}$$

WHEN READY

Grid method

TU x U 23 x 8

Children will approximate first

x	20	3
8	160	24

$$\begin{array}{r} 160 \\ + 24 \\ \hline 184 \end{array}$$

or

x	8
20	160
3	<u>24</u>
	<u>184</u>

72 x 38

Children will approximate first.

72 x 38 is approximately 70 x 40 = 2800

x	70	2
30	2100	60
8	560	16

$$\begin{array}{r} 2100 \\ + 560 \\ + 60 \\ + 16 \\ \hline 2736 \\ 1 \end{array}$$

MOVING ON TO

As soon as children have a secure understanding of grid method, they should move onto

Expanded method

TU x U 27 x 4 (is approximately 30 x 4 = 120)

$$\begin{array}{r} 27 \\ \times 4 \\ \hline 80 \quad (20 \times 4) \\ \underline{28} \quad 7 \times 4 \\ 108 \end{array}$$

HTU x U 346 x 9

346 x 9 is approximately 350 x 10 = 3500

$$\begin{array}{r} 346 \\ \times 9 \\ \hline 12^1 700 \quad 9 \times 300 \\ + 360 \quad 9 \times 40 \\ + \underline{54} \quad 9 \times 6 \\ \hline 3114 \end{array}$$

TU x TU 72 x 38

Children will approximate first. 72 x 38 is approximately 70 x 40 = 2800

$$\begin{array}{r} 72 \\ \times 38 \\ \hline 2^1 100 \quad 30 \times 70 \\ 60 \quad 30 \times 2 \\ 560 \quad 8 \times 70 \\ \underline{16} \quad 8 \times 2 \\ \hline 2736 \end{array}$$

FINALLY

Short multiplication:

4346 x 8

4346 x 8 is approximately 4346 x 10 = 43460

$$\begin{array}{r} 2^4 3^4 4^4 6 \\ \times \quad \quad 8 \\ \hline 34768 \end{array}$$

Long Multiplication

(Multiplication by more than a single digit)

$$372 \times 24$$

Children will approximate first. 372×24 is approximately $400 \times 25 = 10000$

$$\begin{array}{r} ^{12}372 \\ \times \quad ^124 \\ \hline 1^1488 \\ \quad 7440 \\ \hline \quad \quad 8928 \end{array}$$

Using similar methods, they will be able to multiply decimals with one decimal place by a single digit number, approximating first. They should know that the decimal points line up under each other and use place value to help them (whole numbers and \div by 10).

e.g. 4.9×3

Children will approximate first

$$4.9 \times 3 \text{ is approximately } 5 \times 3 = 15$$

$$\begin{array}{r} \times \quad 4 \quad 0.9 \\ 3 \quad \boxed{12} \quad \boxed{2.7} \end{array}$$

$$\begin{array}{r} 12 \\ + \quad 2.7 \\ \hline 14.7 \end{array}$$

$$3 \times 9 = 27 \text{ then } \div 10 = 2.7$$

Moving to efficient written method

$$\begin{array}{r} ^24.9 \\ \times \quad 3 \\ \hline \underline{\underline{14.7}} \end{array}$$

Often, relating decimals to money can help understanding:

$$£4.90 \times 3$$

$$£4.00 \times 3 = £12.00$$

$$90\text{p} \times 3 = \underline{\underline{£ 2.70}}$$

$$£ 14.70$$

Bar modelling is particularly useful when solving multi-step word problems.

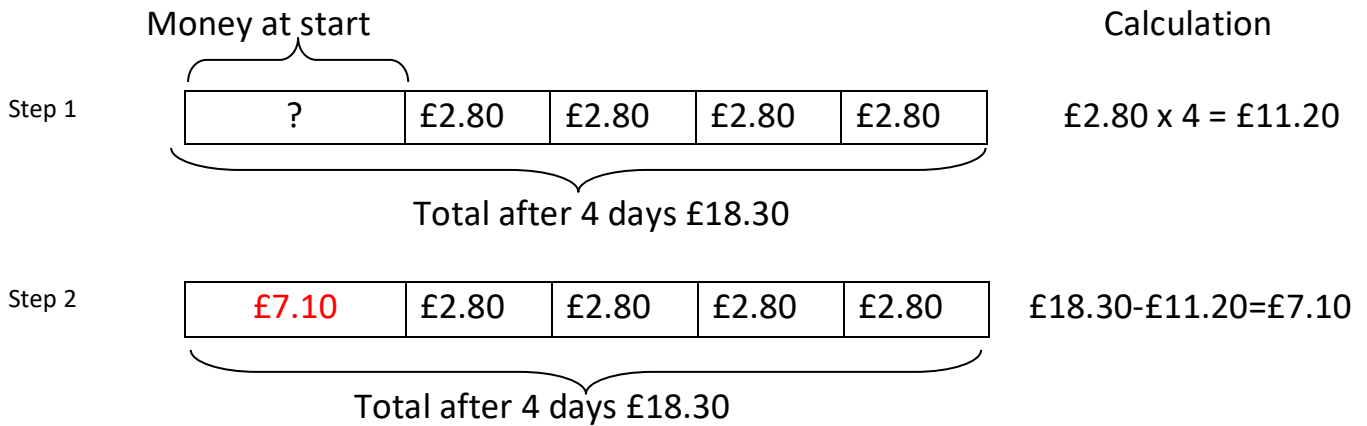
For example:

Flo has some money in a money box.

Each day she puts £2.80 into her money box.

After 4 days she had £18.30 in her money box.

How much money did she have in her money box at the start?



Flo had £7.10 in her money box at the start.